# Robustness of Cosmological Simulations

*Salman Habib (T-8), Katrin Heitmann (ISR-1), Paul Ricker (University of Chicago at Urbana), and Michael Warren (T-6); habib@lanl.gov*

The gravitationally-driven evolution of cold dark matter (CDM) dominates the formation of structure in the universe over a wide range of length scales, from the very longest to subgalactic scales. While the longest scales can be treated by perturbation theory, a fully quantitative understanding of nonlinear effects requires the application of large-scale particle simulation methods. Significantly, precision predictions for next-generation observations, such as weak gravitational lensing, can only be obtained from precise, error-controlled, numerical simulations. The point of the present work is both to characterize the current state of the art and, as a first step, to determine the obstacles that need to be overcome before simulations can satisfy the requirements posed by future observations. The second step is, of course, to develop and provide the physical, mathematical, and computational tools needed to overcome these obstacles.

In the cosmological "standard model" the evolution and structure of the mass distribution is dominated by dark matter, an as yet undetected type of matter that interacts only gravitationally. In order to study spatial and temporal aspects of the mass distribution today, it is convenient to consider three length scales: very large length scales where a linear treatment of the instability suffices, an intermediate length scale where nonlinearities cannot be neglected yet the dynamics of baryons are not of significant importance, and a high-resolution regime where gas dynamics, star formation, etc., need to be taken into account. The transition from the linear regime to the nonlinear regime today (redshift $z = 0$) occurs at length scales of order several Mpc (or wave number $k \sim 0.1$ Mpc$^{-1}$). Galaxies exist within dark matter halos, and typical halo size scales are

$\sim 0.1$–1 Mpc. Thus, in order to resolve the dynamics of halos, a spatial resolution $\sim 10$ kpc appears to be necessary.

Our convergence and comparison study of cosmological simulations focuses on the dynamics of the cold dark matter [1]. Given the importance of this component and expected precision observations from large-scale structure and lensing observations, this study is timely and important. We have focused on medium resolution simulations, i.e., where the spatial resolution is $\sim 10$–100 kpc. These simulations are adequate for matter power spectrum computations, for weak lensing, and for gathering cluster statistics. Planned observations of these quantities promise control of systematic and statistical errors at the few to 1% level, thus it is crucial that confidence in the codes be established to better than this level of uncertainty. Even though numerical codes have advanced considerably both in terms of performance and error control, this is by no means an easy task.

Six different codes have been utilized in our work. These represent a broad sampling of the present state of the art: the particle-mesh code MC$^2$ (Mesh-based Cosmology Code); the adaptive-mesh-refinement (AMR) code FLASH; the tree code HOT (Hashed-Oct Tree); the tree code GADGET (GAlaxies with Dark matter and Gas intEracTions); TPM, a tree particle mesh code, and HYDRA, an adaptive-particle-particle-mesh code. (MC$^2$ and HOT were developed at Los Alamos National Laboratory.) Since all the codes have quite different algorithms and differing error modes, convergence to a single solution is a strong test of the validity of the N-body approach in modeling dissipationless gravitational dynamics at the resolution scales probed by the tests. All codes were run with identical initial conditions: the initial conditions and final results are publicly available.

A variety of tests and diagnostic tools have been employed, beginning with the Zel'dovich pancake test to investigate issues of convergence and collisionality. As a result of these and other tests, we concluded that unphysical collisionality was not a problem with code accuracy, as had been

earlier speculated by some authors. We then considered two more realistic situations, the test initial condition known as the "Santa Barbara cluster" and LambdaCDM simulations with cosmological parameters as given by the latest results from the WMAP satellite and other cosmic microwave observations as well as large-scale structure observations from the Sloan Digital Sky Survey. A very complete set of diagnostic tools such as velocity distributions, mass functions, power spectra, correlation functions, etc., were used to compare code results.

Over the range of mass and force resolutions that we deemed to be reasonable for comparison purposes, the codes agreed with each other at roughly the 5% level. This result is encouraging, but much work needs to be done. In the next several years, code accuracy has to be improved by an order of magnitude. We hope that work such as ours will help pinpoint the areas needed for improvement and lead the community effort in improving the state of the art of cosmological simulations.

We now provide two illustrative examples of the sorts of difficulties discovered during the tests. The first relates to the calculation of the distribution of dark matter halo masses, the so-called halo mass function. The mass function of galaxy clusters provides a useful probe of the matter density and dark energy equation of state. We have found that AMR codes require special handling of the initial
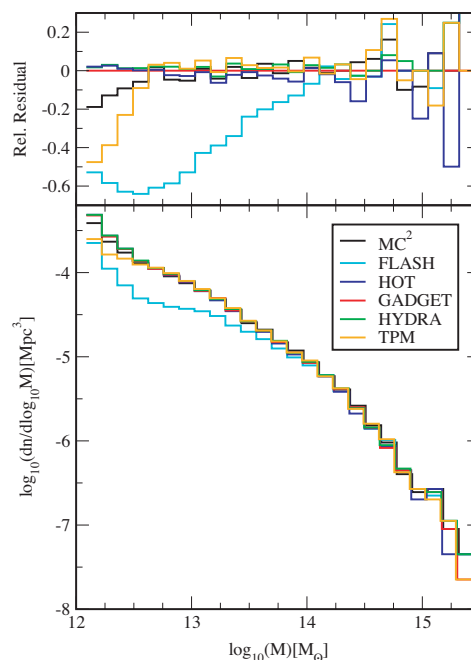
conditions in order to not lose small mass halos (Fig. 1). The second is a comparison of the projected mass density of the Santa Barbara cluster. While the overall agreement for the density profile is very good, small features in the different code results do not agree, even though these features are larger than the formal force resolutions employed (Fig. 2).

[1] K. Heitmann, P.M. Ricker, M.S. Warren, and S. Habib, Los Alamos National Laboratory report LA-UR-04-5954 (2004), astro-ph/0411795.
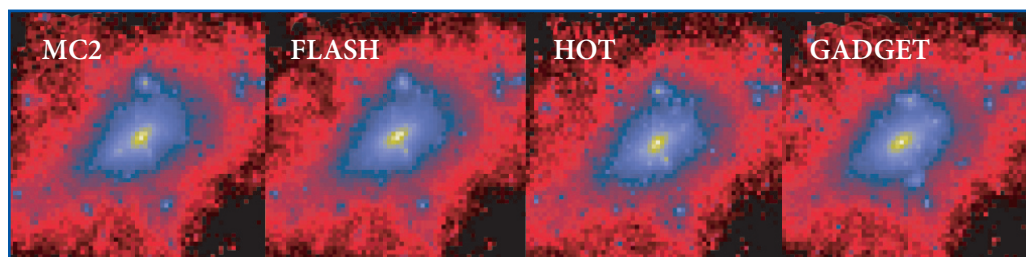
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Figure 2—
The projected mass density for the inner 8 Mpc of the Santa Barbara cluster run with 17 million particles. The pixel size is 62.5 kpc (each axis is 128 pixel lengths); no smoothing was employed. Note the lack of complete feature-by-feature agreement on pixel scales though the overall morphological agreement is very good. The formal resolutions for the codes were as follows: MC$^2$, 62.5 kpc; FLASH, 62.5 kpc; HOT, 5 kpc; GADGET, 50 kpc.

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